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Uncovering CDM halo substructure with tidal streams

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ABSTRACT

Models for the formation and growth of structure in a cold dark matter dominated universe predict that galaxy halos should contain significant substructure. Studies of the Milky Way, however, have yet to identify the expected few hundred sub-halos with masses greater than about $10^6\,M_\odot$. Here we propose a test for the presence of sub-halos in the halos of galaxies. We show that the structure of the tidal tails of ancient globular clusters is very sensitive to heating by repeated close encounters with the massive dark sub-halos. We discuss the detection of such an effect in the context of the next generation of astrometric missions, and conclude that it should be easily detectable with the GAIA dataset. The finding of a single extended cold stellar stream from a globular cluster would support alternative theories, such as self-interacting dark matter, that give rise to smoother halos.

1 INTRODUCTION

A generic prediction of Cold Dark Matter (CDM) cosmology is the existence of a significant amount of substructure in gravitationally collapsed structures such as galaxy clusters, galaxy halos and even dwarf galaxies (Klypin et al. 1999; Moore et al. 1999b, 2001). On the scale of large spiral galaxies like the Milky Way, some 500 dense clumps are expected to orbit in the Halo. These structures have very dense cores, with a very steep radial profile of universal shape (increasing as r^{-1} or $r^{-1.5}$ in the central regions, Navarro, Frenk & White 1997; Moore et al. 1999a), which renders them essentially impervious to Galactic tides.

The current "dark matter crisis" stems from the difference between these predictions and observations of low-mass galaxies. The first problem is that rotation curves of gas-rich low-mass disk galaxies are not as peaked as required by CDM. Van den Bosch et al. (2000) suggested that this disagreement could be due to an error in the analysis of HI observations, in which the beam-smearing effect of extant HI radial velocity curves had not been taken into account. However, this appears not to be the case, as recent (high spatial resolution) H_α rotation curves have shown (Côté et al. 2000; de Blok et al. 2001; Marchesini et al. 2001).

A second fundamental problem also exists: the mass of the CDM substructures in a galaxy-sized halo is similar to that inferred for galaxy satellites, yet the number of CDM substructures in a galaxy-sized halo is much larger than the number of observed dwarf galaxies(Klypin et al. 1999; Moore et al. 1999b). It is, of course, possible that these galaxies have a quite different non-cosmological origin (perhaps due to tidal interactions in the early universe). In either case the problem remains: where is the population of 500 CDM substructures orbiting the Milky Way and other large galaxies? Recently, several studies have investigated the effect of reioninzation on the early evolution of small structures (Bullock et al. 2000; Somerville 2001; Tully et al. 2001). This re-ionizing radiation sets in at a redshift ~ 6–10, and originates from either the first stars or quasars. Indeed, the first direct evidence for reionization (at $z \sim 6$) has recently been found (Becker et al. 2001). The intergalactic and galactic mediums are ionized by this radiation, and any gas that is not in deep potential wells is lost from the protogalaxies. The limiting mass for maintaining a gas component appears to correspond to a circular velocity of $\sim 30\,\mathrm{km\,s^{-1}}$, well above the circular velocity of all the dwarf spheroidal galaxies known. (However, it may be possible for low-mass systems to maintain a gas-fraction if the gas managed to cool to dense molecular form before the epoch of reionization). This "squelching" of star-formation in low-mass galaxies neatly solves the CDM satellite over-production problem.

A potential candidate for the missing dwarf systems that are supposed to inhabit the Galactic halo is the large population of high velocity clouds that pepper the sky (see Wakker 2001 for a recent review). With little or no stellar content, these are seen to possess velocities incompatible with Galactic rotation models, although Blitz et al. (1999) have shown that the kinematic characteristics of the velocities of the clouds are compatible with them being members of an accreting population distributed within a megaparsec of the Milky Way. At these distances, the kinematic signatures of the clouds imply a mass-to-light ratio of 10-50 Braun & Burton (2000). Currently, their distances are extremely difficult to determine, with Complex A being the only system to which the distance, of between 4 and 10kpc, has been measured (van Woerden et al. 1999). With the promise of accurate distances from H_α (Bland-Hawthorn & Maloney 1999), a full appraisal of the nature of high velocity clouds and their relation to the formation of the Milky Way

can be made; but a preliminary analysis suggests that the clouds are scattered in the Galactic halo (Bland-Hawthorn & Maloney 2001), and are not of cosmological origin.

Nevertheless, CDM theory predicts that a large population of completely or almost completely dark "galaxies" should be present in the Milky Way. Here we propose an observational test of this scenario. In the early galaxy, many globular clusters are expected to have surrounded the Milky Way. Galactic tidal forces have been destroying these objects (Gnedin & Ostriker 1997), and of the initial population of a few hundred objects, only $\sim$ 100 remain, and many of these show signs of tidal disruption (Leon, Meylan & Combes 2000; Odenkirchen et al. 2001).

In a static potential, as a low-mass stellar system, such as a globular cluster, loses progressively more and more mass to tidal forces, it develops long tidal tails that closely follow the orbit of the globular cluster. (In the limit of a zero mass system, the tidal stream exactly follows the progenitor orbit). Here we explore how the presence of the population of dark matter clumps predicted by CDM alters the phase-space structure of a globular cluster tidal stream. For the special case when the global Galactic potential is nearly spherical, this corresponds to a broadening of the stream from a thin great-circle stream into a wide band on the sky. In a companion paper (Ibata et al. 2002; Paper 2), we use these results to motivate the search for tidal streams in the 2 Micron All Sky Survey (2MASS) dataset.

Other ways to probe the dark matter substructure have been proposed. Recently, Metcalf (2001) and Chiba (2001) suggested that substructure in external galaxies may be detected through their gravitational lensing of background quasars, as a lumpy halo will produce image and brightness configurations different to a smooth matter halo. While such an approach may reveal any missing sub-halos, the method is subject to the vagaries of gravitational lens modeling. During the final preparation of this work, Mayer et al. (2001) presented a study investigating the signatures that are imprinted by different dark matter models on tidal streams of dwarf spheroidal galaxies. They find that the structure and kinematics of the outer regions of dwarf galaxies can also allow one to differentiate between dark matter models.

2 NUMERICAL SIMULATIONS

Our aim is to quantify the difference in the structure of the tidal tail of an ancient globular cluster (or even an ancient globular cluster remnant) if the Galactic halo has a smooth mass distribution compared to the case when the Halo also contains significant substructure. We have modelled the smooth components of the Galaxy as fixed potentials, using the Galactic mass model of Dehnen & Binney (1998), which contains a disk, thick disk, interstellar medium, bulge, spheroid and halo components. The parameters of this model are detailed in Ibata et al. (2001). The halo was taken to have a mass normalization that gives a total model circular velocity of $v_c = 210\,\mathrm{km\,s^{-1}}$ at 50 kpc. The mass flattening of the potential was left as a free parameter q_m.

We adapted the fast parallel code PKDGRAV to include the forces due to these mass distributions by including a multipole expansion code kindly supplied by W. Dehnen. In all the following simulations we maintained accurate forces by using an opening angle of 0.75 and expanding the cell moments to hexadecapole order. Two body relaxation was suppressed by using a spline softening of 10 pc, such that the inter-particle forces are completely Newtonian at 20 pc. A variable time-step scheme is used based on the local acceleration, $\Delta t < \eta\sqrt{|\Phi|}/a$, and density, $\Delta t < \eta/\sqrt{G\rho}$, with the accuracy parameter $\eta = 0.03$ (Quinn et al. 1997). Typically we used 100000 base steps to integrate 12 Gyr. With the variable time-steps, this was equivalent to taking 5×10^7 time-steps.

The initial globular cluster was modelled with a King model (King 1966), populated with 10^4 particles. The total mass of the globular cluster was taken to be $10^6\,\mathrm{M_\odot}$, the model had concentration parameter $c = 1.0$ and central velocity dispersion $\sigma = 4.1\,\mathrm{km\,s^{-1}}$, yielding a tidal radius of $r_t = 300\,\mathrm{pc}$. This model is of approximately the same size and mass as Omega Centauri, but of lower concentration (by a factor of 1.7). Note that we are not aiming at presenting an accurate model of the disruption of a globular cluster — we do not have sufficient resolution for that — instead, here we seek to study the kinematic behaviour of the unbound tidal streams that emerge from disrupting low-mass systems.

The initial position and systemic velocity of the King model were randomly chosen from the halo component of a galaxy model (Boily, Kroupa & Penarrubia-Garrido 2001) that gives a similar rotation curve to the adopted Dehnen & Binney mass model.

Integrating the globular cluster models for 10 Gyr in the smooth potential gives rise to very narrow tidal tails. An example of one of these models is shown in Figure 1, which displays the position, heliocentric radial velocity and distance distribution of the stream particles. At its narrowest, the width of this stream is $\sim$ 100 pc, similar in width to the tidal radius of the initial King model, while at its widest, the stream is $\sim$ 3 kpc wide. The shape of the halo mass distribution for the simulation displayed in Figure 1 is spherical, $q_m = 1$, which is motivated by recent analyses of the outer halo of the Milky Way (Ibata et al. 2001; Chiba & Beers 2001).

For each one of these models, we also investigate the effect of the presence of a large number of moving substructures in the halo. However, we first need a realistic way of modeling these substructures. According to Navarro, Frenk & White (1997) (hereafter NFW), and other authors, galaxy halos have a "universal" density profile $\rho(r) \propto 1/(r(1+r)^2)$, which fits all scales currently probed by the numerical cosmology simulations. The potential corresponding to this density profile is very simple: $\phi(r) \propto 1 - \ln(1 + r)/r$, and the forces due to this potential would be straight-forward to add in to our N-body integrator. Unfortunately, however, this density profile is not well-behaved either at small or at large radii (the total mass diverges); to fix these problems, the force near the center of the sub-halo would need to be softened, and the density would have to be made to fall off to zero at some large radius (the tidal radius of the substructure would be a good choice). A simpler solution is to approximate the NFW profiles with a softened point-mass potential. Figure 2 compares the radial acceleration due to an NFW potential (dotted line), a Kepler potential (dashed line), and a softened Kepler potential (solid line) with spline softening of 3.41 times the scale radius r_s of the NFW pro-

file. Note that, at all radii, this softened point-mass potential underestimates the forces compared to the NFW potential.

The substructures are added into the Galactic halo as softened point masses, with a distribution of sub-halo masses that follows a relation $\log_{10}(N_c) = 3(1 - 4v_c/V_{Global})$ (our parameterization of Figure 2 of Moore et al. 2001). At the low-mass end of this distribution, there are 435 sub-halos of circular velocity $v_c > 0.03V_{Global}$ ($M \sim 10^7\,\mathrm{M}_\odot$). The halo mass fraction in this lumpy component is relatively small, as it accounts for less than 10% of the mass of the smooth halo. The initial positions and velocities of the particles representing the sub-halos are also chosen from the halo component of the Boily, Kroupa & Penarrubia-Garrido (2001) Galaxy model, but with a pericenter cutoff of 10 kpc, since dynamical friction will likely make the orbits of dark satellites with smaller pericenter radii decay quickly.

The effect of these numerous dark matter clumps is marked: in Figure 3 we show the position, velocity and distance distribution of the tidal stream particles. In this simulation, only 1% of the mass of the halo is not in the smooth component, yet the tidal stream has been substantially heated and is substantially wider than in the smooth halo (the stream is so fluffy that it is difficult to measure a width).

A better way to look at the simulations is in the space of the integrals of the motion. In the axisymmetric potentials considered here, the integrals are the total energy per unit mass E and the z component of angular momentum per unit mass L_z. It is also useful to inspect the total angular momentum per unit mass, L, which is an approximate integral of the motion in the relatively spherical potentials we consider here. Figure 4 shows the relation between these three quantities. The top row of Figure 4 is derived from the simulation of the globular cluster model in the smooth spherical halo whose structure on the sky was previously displayed in Figure 1. After 12 Gyr the particles have remained localised in the space of the integrals of the motion. The bottom panel shows the same globular cluster model on the same orbit, but simulated in the halo model that contains the additional NFW sub-halos (corresponding to the simulation shown in Figure 3). In this lumpier potential, the globular cluster stream is comparatively much more dispersed, especially in the L_z parameter. The r.m.s. dispersion in L_z is a factor of 5 larger in the lumpy halo simulation.

Inspecting the spatial distribution of particles can be useful for differentiating between a smooth and lumpy halo when the mass distribution is spherical (or close to spherical). However, in a flattened halo, differential precession mimics the effect of heating by CDM clumps, and it becomes impossible to differentiate between the two causes of spatial dispersion of the stream. It is only when viewing the space of the integrals of the motion that the effect of precession decouples from the stochastic heating by the CDM clumps. We have repeated the same experiments as above in a flattened halo, with $q_m = 0.7$, a flattening value predicted by numerical cosmology (Katz 1991; Dubinski & Carlberg 1991; Warren et al. 1992; Katz & Gunn 1991; Summers 1993; Dubinski 1994). The sub-halos are chosen from a halo distribution function which is also flattened to $q_m = 0.7$. Figure 5 shows the relation between the integrals of the motion E and L_z and the approximate integral L. Again, the dispersing effect of the sub-halos is very strong, with the r.m.s. dispersion in L_z increasing by a factor of 9 with respect to the smooth halo case.

2.1 Suite of simulations

To obtain a statistical understanding of the change in L_z, we undertook a series of simulations of the globular cluster model with initial position and velocity randomly-drawn from the spherical halo distribution function model mentioned above. Ten simulations were undertaken in the Galaxy model with the smooth spherical halo. A further ten simulations probed the same models with the addition of the NFW sub-halos, for a total of 20 simulations with $q_m = 1.0$. To probe the situation with a more flattened halo, we undertook a further 20 simulations with $q_m = 0.7$.

The result of these experiments is shown in Figure 6, where we display the r.m.s. dispersion in L_z for simulations in a smooth halo (filled circles) and in in the presence of halo substructures (star symbols). The top panel shows the case in a spherical halo ($q_m = 1.0$), while the bottom panel is for the flattened halo ($q_m = 0.7$) simulations.

Both in a spherical and in a flattened halo, the presence of CDM-like substructures increases substantially, on average, the RMS dispersion in L_z of the stream stars. On some orbits (such as that of our simulation #9, the effect can be small, however. Our simulations suggest that globular cluster streams with a very dispersed specific angular momentum, say $\sigma L_z > 200\,\mathrm{kpc\,km\,s^{-1}\,M_\odot}^{-1}$ require that the halo possesses significant substructure. Smaller values, $\sigma L_z < 50\,\mathrm{kpc\,km\,s^{-1}\,M_\odot}^{-1}$ say, would require a smooth halo.

3 OBSERVATIONAL CONSIDERATIONS

We next consider whether it will be possible to identify a $10^6\,\mathrm{M}_\odot$ stream orbiting the Halo at distances up to 100 kpc ($m - M = 20$) with photometry, radial velocity, parallax and proper motion data from the future astrometric mission GAIA (Perryman et al. 2001). The great advantage of the GAIA dataset is that gives access to the full 6-dimensional phase space information for many stars, and at least 4-dimensional phase-space information (for those stars that are too far to have measured parallaxes and radial velocities). In addition, the photometric information (in 15 bands) can give age and/or metallicity estimates, providing a powerful discriminant between stellar populations.

Developing a full model of the Galaxy as viewed by GAIA is not a trivial task and it is beyond the scope of this paper. Here we simply make an order of magnitude estimate of the contrast of plausible globular cluster streams over the background by estimating the relative phase space density of stream stars over the large-scale Galactic spheroid (or stellar halo) component. We estimate the phase-space size of the stream as follows. The $\sim 10\,\mathrm{km\,s^{-1}}$ radial velocity uncertainty of GAIA exceeds the intrinsic radial velocity dispersion of the stream, and is about 10% of the halo radial velocity dispersion. The proper motion dispersion at 100 kpc corresponds to about $\sim 10\,\mathrm{km\,s^{-1}}$ or 10% of the halo dispersion (two coordinates). The positional width of the streams are $\lesssim 3$ kpc, or about 1% of the width of the halo. With the

full 15 photometric bands, the distance uncertainty of photometric parallax measurements is likely to be $< 10\%$. Thus the phase-space volume occupied by the stream is likely to be about one millionth of that of the halo. Since the stellar halo has a mass of only $\sim 10^9\,\mathrm{M}_\odot$, the contamination will amount to approximately one star in a thousand. With a limiting magnitude of $V \sim 21$, the GAIA measurements will easily reach down to the horizontal branch at 100 kpc. Placing all of the stream at a distance of $d = 100$ kpc, we expect ~ 1600 stars for $V > 21$ per $10^6\,\mathrm{M}_\odot$ of progenitor mass (assuming the luminosity function of the Draco by Odenkirchen et al. 2001, corrected to a mass to light ratio of $M/L = 3$). For radial velocity measurements the limiting magnitude is $V \sim 17$, and the number of available stream stars from the $10^6\,\mathrm{M}_\odot$ population drops to ~ 180. The detection of these streams will be straightforward with GAIA, and the increase in the dispersion of L_z due to any halo substructure will be easily resolved.

With results expected towards the end of the second decade of this century, the GAIA mission is not for the impatient. So it is worthwhile to investigate whether any presently available datasets already contain sufficient information to uncover the predicted Galactic halo dark substructures. To this end we undertook an analysis of the 2MASS dataset, which provides positions and infrared CCD photometry for sources brighter than $K = 14.3$. This gives access to a relatively restricted sample of Galactic halo M-giants, with no information on their kinematics, and only a very poor constraint on their distances. The search for the halo streams in the 2MASS dataset is presented in Paper 2; the analysis reveals a single very strong stream, that due to the ongoing disruption of the Sagittarius dwarf galaxy.

Is it possible to use the Sagittarius stream to constrain the lumpiness of the halo? To investigate this issue we ran two simulations, in the same manner as above, one in a smooth halo, and the other with NFW clumps, using as an initial model for the Sagittarius dwarf the model D2 of Ibata et al. (2001). Figure 7 shows the result of this experiment. This model is much more massive ($M = 5 \times 10^8\,\mathrm{M}_\odot$) than the globular cluster model simulated above, and is initially much more extended (half-mass radius of 0.9 kpc), so it is naturally widely dispersed in E, L_z and L. Though the stream particles are heated and dispersed by the sub-halos, the effect is subtle, and will be very difficult to disentangle from the normal dynamical evolution of the dwarf galaxy. So low mass streams from globular clusters need to be found to act as probes of the dark matter.

4 CONCLUSIONS

We have shown that stellar streams of low mass systems such as disrupted globular clusters can be used to constrain the substructure of the dark halo. If current cosmological simulations correctly represent reality, ancient stellar streams from globular clusters should be substantially dispersed over the sky, even if on average, the Galactic halo potential is close to spherical. This effect is readily noticeable, as the streams will have a large dispersion in the z-component of specific angular momentum L_z. For the particular halo studied in this contribution $\sigma L_z > 200\,\mathrm{kpc\,km\,s^{-1}\,M_\odot}^{-1}$ implies the presence of substantial dark matter substructure.

We have simplified the analysis in two major ways. Highly softened point-masses were used to model the halo substructures, which leads to an underestimate of the increased dispersion in L_z in the presence of the NFW clumps. Our other major assumption, that the Galactic potential is axisymmetric and does not evolve with time, will likely have the opposite effect, namely to underestimate the dispersion in the absence of the NFW clumps. Our study should be refined once the Galactic potential is better constrained (with data from the GAIA and SIM missions).

This work has shown that an all-sky survey with high precision astrometry, such as will be obtained with the GAIA satellite, we will easily be able to detect globular cluster streams, and measure their kinematic properties, especially the dispersion in L_z.

The heating due to the numerous encounters with the dark substructure must also affect the computed destruction rate of globular clusters and low-mass dwarf galaxies alike. It is possible that the lower limit to the mass of dSph galaxies $\sim 10^7\,\mathrm{M}_\odot$ may be set by these disruptive encounters.

Discovering and determining the thickness of ancient stellar streams will allow us to probe the invisible dark halo structures predicted by Cold Dark Matter cosmology. Finding a single narrow stream of width comparable to the tidal radius of a globular cluster would stand in strong contradiction with this theory. Data from the forthcoming GAIA mission planned by the European Space Agency will provide the opportunity to undertake this study.

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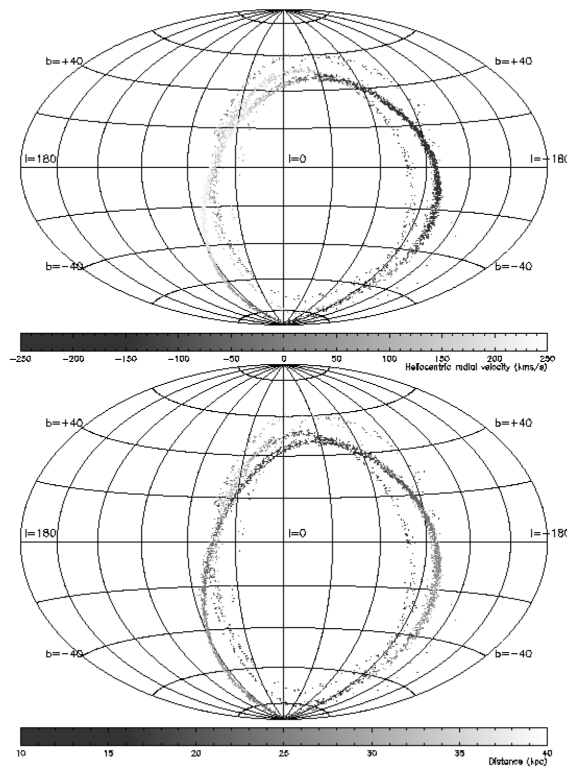

Figure 1. The sky distribution (in Galactic coordinates) of one of the globular cluster models after 12 Gyr. The Galactic potential has been modelled as a sum of a disk, thick disk, interstellar medium, bulge and spheroid components, plus a spherical dark halo. In this simulation all of these Galactic components have potentials that are smooth, axisymmetric, and static. The upper diagram shows the particle velocities and the lower diagram shows the particle distances.

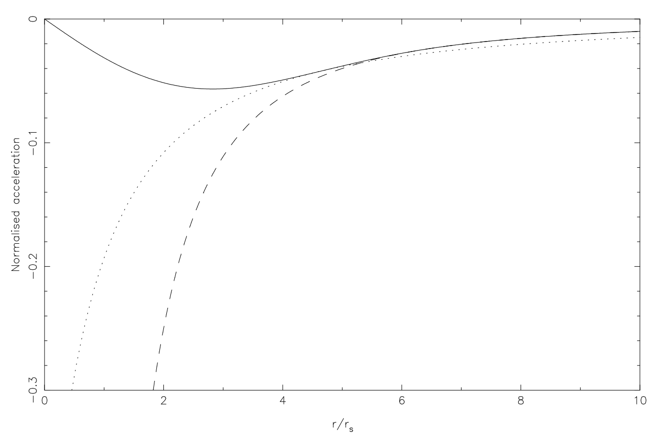

Figure 2. The acceleration as a function of radius due to an NFW halo (dotted line) a point-mass (dashed line) and a spline-softened point mass with $r_{soft} = 3.41 r_s$. By modeling the NFW sub-halos as softened point masses, we underestimate the forces (and hence the heating) they impart on other particles in the simulation.

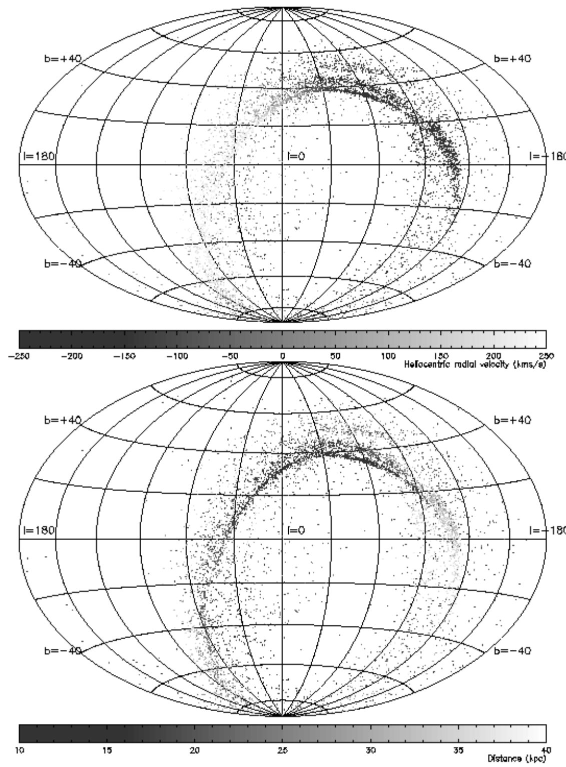

Figure 3. As Figure 1, but this time the globular cluster has been simulated in a Galactic potential replacing a small mass fraction of the halo with 435 moving NFW sub-halos.

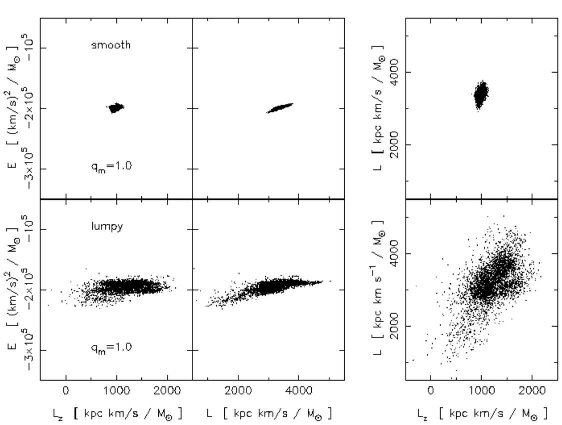

Figure 4. The relation between the integrals of motion for the simulation displayed in Figure 1 (top row) and the simulation displayed in Figure 3 (bottom row). The addition of a small fraction by mass in lumpy sub-halos has completely smeared out the stream.

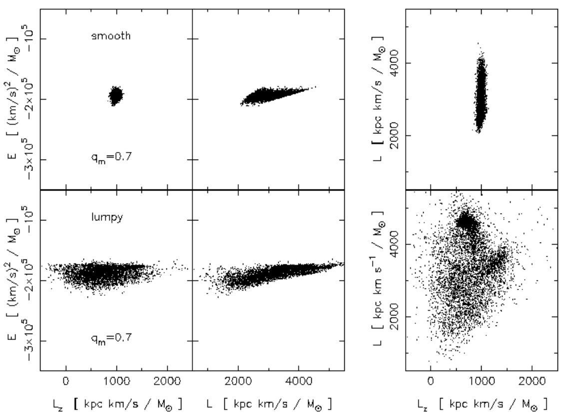

Figure 5. As Figure 4, but in a flattened halo with $q_m = 0.7$. Evidently, even in a flattened halo, it is relatively straightforward to distinguish the effect of the sub-clumps.

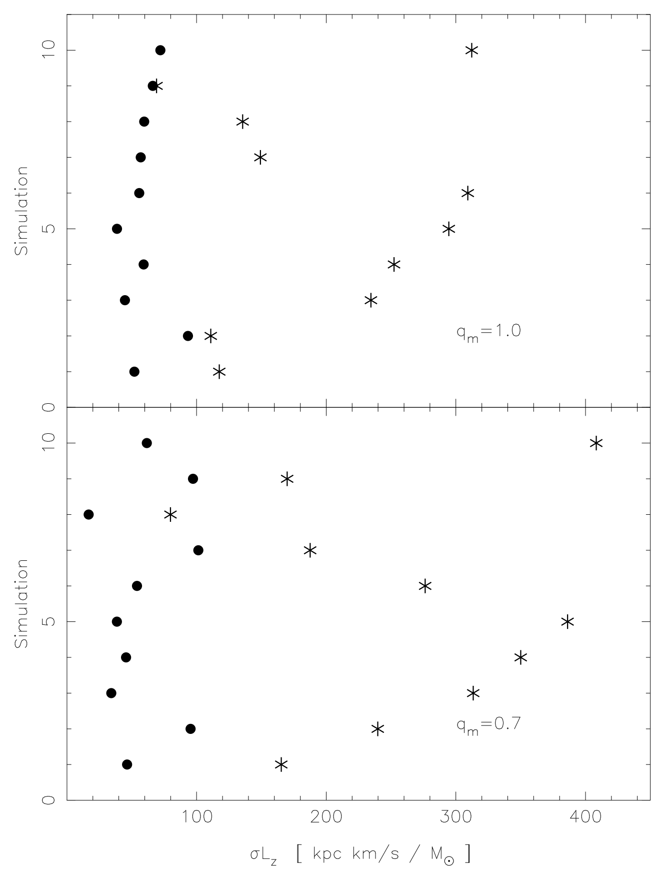

Figure 6. The dispersion in the z-component of angular momentum per unit mass (σL_z) in several simulations. Filled circles denote the smooth halo simulations, while star-symbols show the results with NFW sub-halos. The top panel is for those simulations with a spherical mass distribution, while the bottom panel shows the results in a flattened halo $q_m = 0.7$. Clearly, if the Galactic halo has substantial substructure as predicted by cosmological simulations, globular cluster streams should have a wide dispersion in L_z.

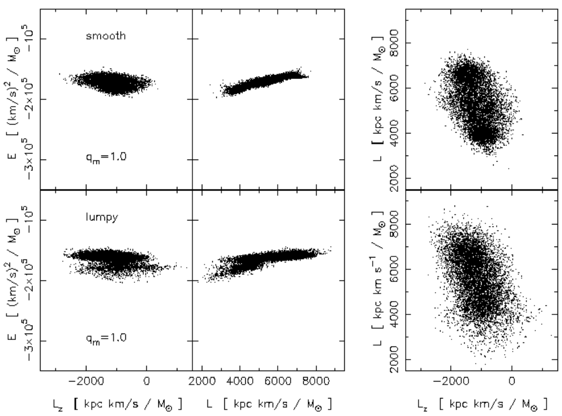

Figure 7. As Figure 4, but for the case of a stream which has already been detected: that of the Sagittarius dwarf galaxy. Here, the difference between the smooth and lumpy halos is much less marked due to the fact that the initial model is much more massive, and has naturally a wide spread in E, L_z and L.